FRACTIONS:
A CONCEPTUAL APPROACH

A Singapore Math Topical Presentation
Grades 3-6

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Three distinct meanings of fractions

- **Part of a Whole**: the fraction $\frac{1}{5}$ indicates that a whole has been separated into **five** equal parts and **one** of those parts is being considered.
- **A Quotient**: the fraction $\frac{2}{3}$ may be considered as a quotient, $2 \div 3$.
- **A Ratio**: a ratio is a *comparison* of two numbers, so the fraction $\frac{5}{8}$ may be viewed as the ratio 5 to 8 (usually written 5 : 8).

Numerator & Denominator

Numerator — Total number of objects (or pieces to be shaded) we actually have

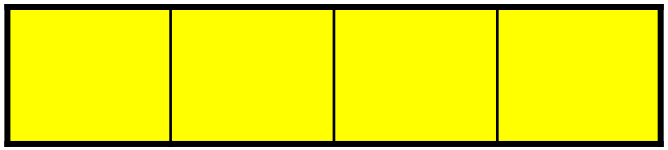
Denominator — Number of <u>equal</u> pieces which a whole or unit was originally divided into

7/4 means 7 parts to a whole of 4 parts

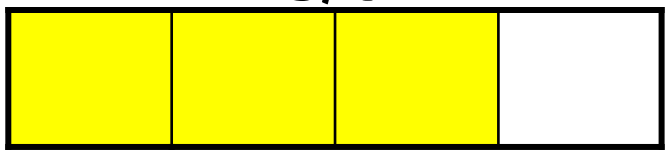
$$\frac{7}{4} \quad \text{or} \quad 1+\frac{3}{4} \quad \text{or} \quad 1\frac{3}{4}$$

Number of pieces in one unit (pie cuts into 4 slices, 1 pie=1 unit)

4/4 (1 unit or pie)

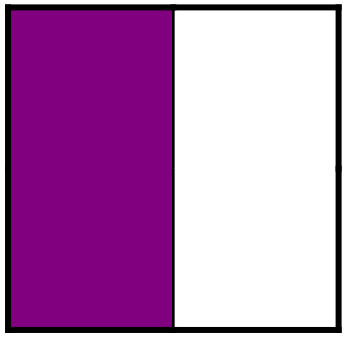


3/4

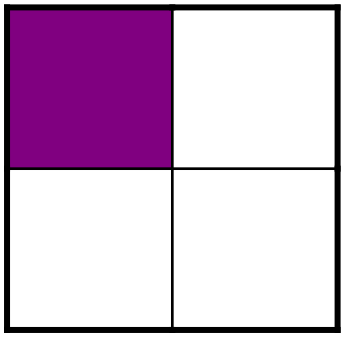


Unlike Fraction

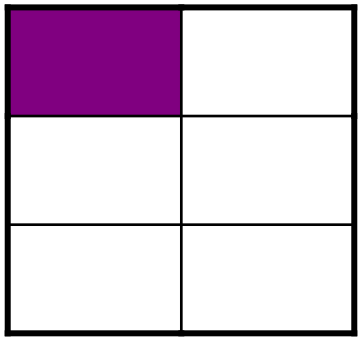
As denominator grew larger what happened to the shaded area of rectangle?



1/2



1/4



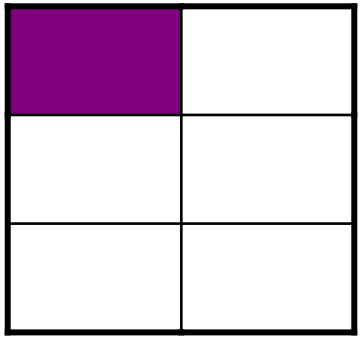
1/6

As the denominator increases the number of pieces the unit is divided into also increases. This means the fraction grows smaller, so each piece of the whole is smaller.

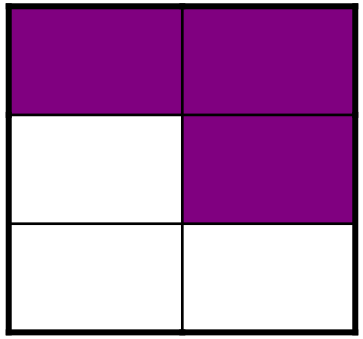
Like Fractions

(Two or more fractions with the same denominator)

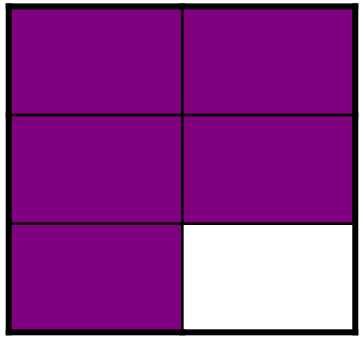
If the numerators of two fractions are different but the denominators are the same, which fraction is larger?



1/6



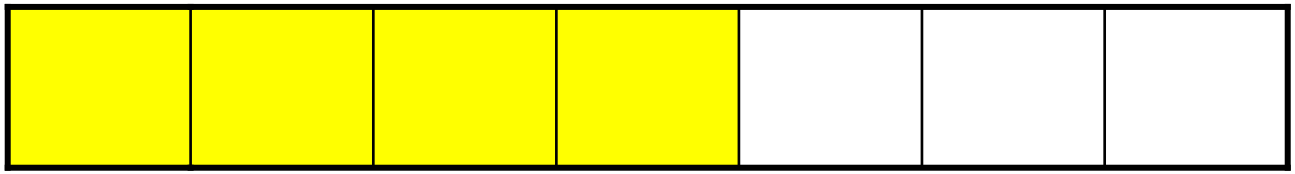
3/6



5/6

Since the denominators are the same, a number of equal parts are being compared. So, 5 is greater than 3, 5/6 must be larger than 3/6.

Fraction Number Sense $\frac{4}{7}$



$\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$

$\frac{4}{7} = 4 \times \frac{1}{7}$ or $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$

Define the Whole

A school has two busses. On the first bus, there are 10 boys out of 40 students. On the second bus, there are 20 boys out of 40 students. What is the fraction of boys to the entire student population?

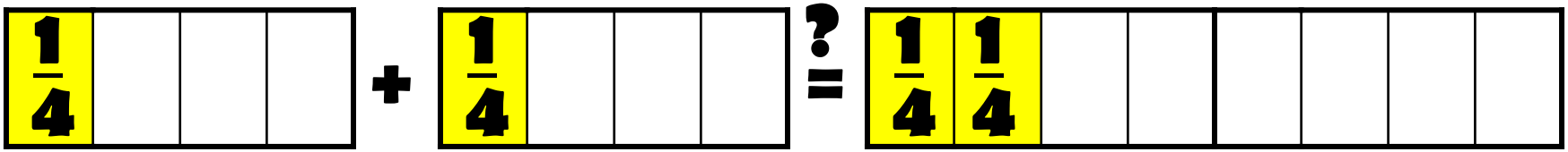
Clay eats **1/4** of a pizza for lunch and **2/4** of a pizza for dinner. What fraction of the pizza did he eat?

Lucy spent a quarter to buy news paper and 50 cents for coffee. How much she spent?

We must establish these clear concepts with our students!

- Do not change the definition of "whole" in the middle of the game.
- A whole can be divided into a different number of equal parts.
- In general, fractions are numbers between 0 and 1.

1/4 + 1/4 DOES NOT GIVE 2/8



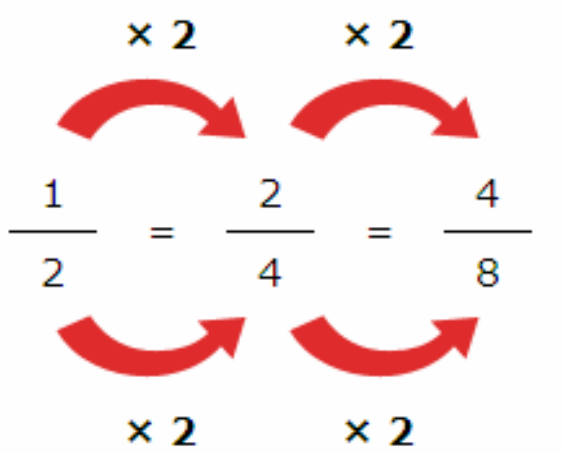
1/4 1/4 2/8 = 1/4

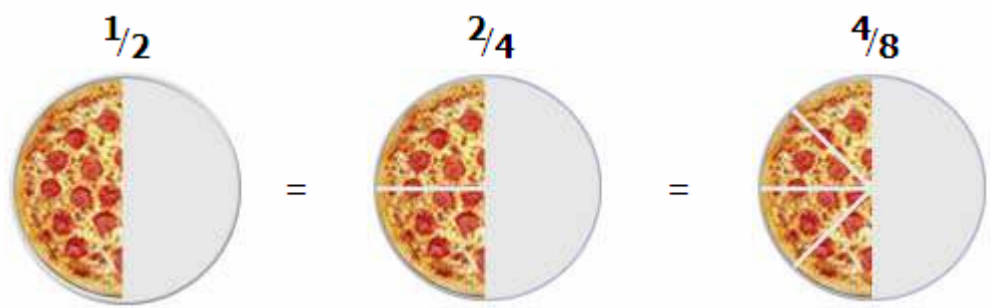
- The left hand side shows two wholes each divided into four equal pieces. The right side shows one whole equal to the original two wholes divided into eight equal pieces. Therefore the same whole or unit is not defined on each side of the equation.
- If you add the denominators (of two Like Fractions) you double the number of pieces needed to make a whole. You are not being consistent.

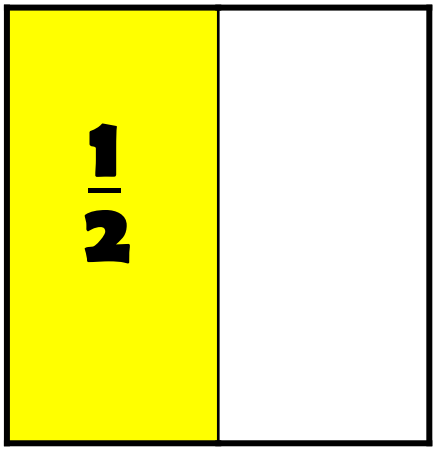
Equivalent Fractions

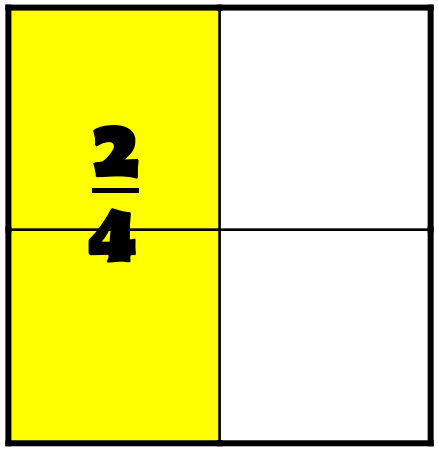
Fractions that have the same value.

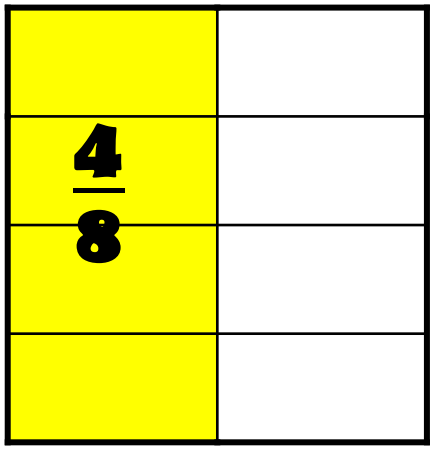
Their numerators and denominators are in the same ratio.







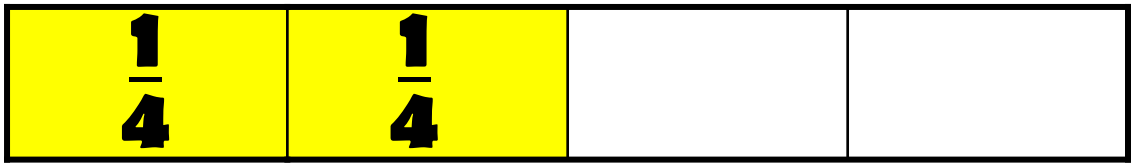




Adding & Subtracting Like Fractions

Adding & Subtracting Like Fractions

Want to add 1/4 + 1/4:



You can see that 1/4 + 1/4 = 2/4 or 1/2

Fraction arithmetic: When the denominators are the same, add the numerators.

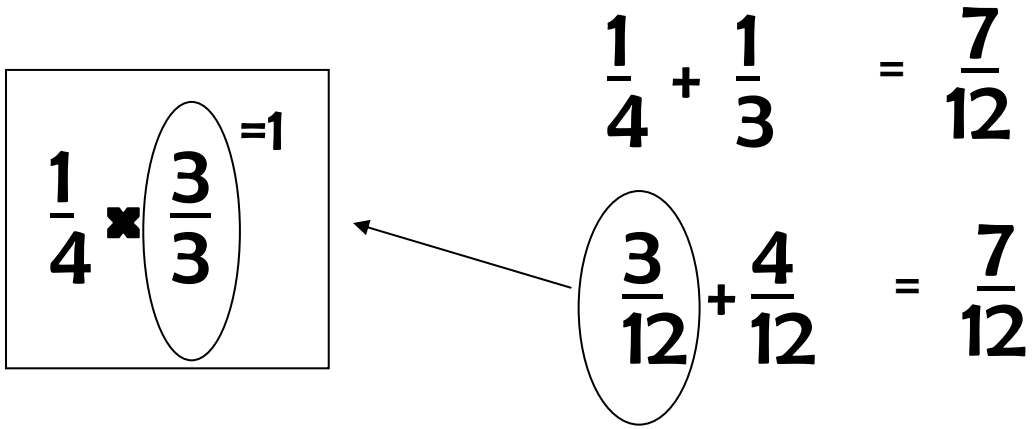
Adding the numerators gives the total number of pieces. This number must be referenced to a standard number of pieces needed to form a whole

$$\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4} \quad \leftrightarrow \quad \frac{1+2}{4}=\frac{1}{4}+\frac{2}{4}$$

Adding & Subtracting Unlike Fractions

How can we add ¼ and 1/3?

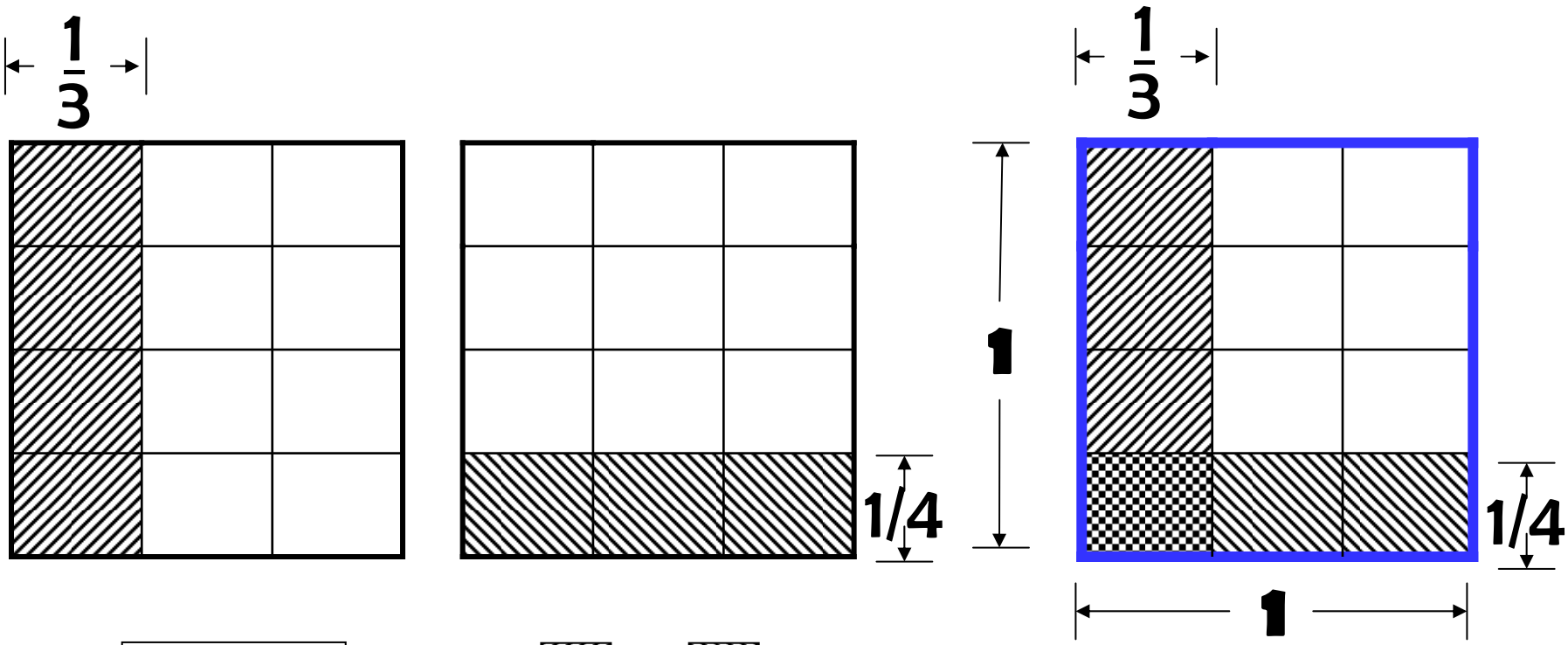
- Relate to something that break easily into third or quarter such as foot, a day, money.
- Consider a foot (12 inches). ¼ of a foot is 3 inches and 1/3 of a foot is 4 inches
- Together, they make 7 inches, which is 7/12 of a foot



Notice: to add these, they had to be written with the same whole!

Addition of Fractions with Grid Model

$\frac{1}{3} + \frac{1}{4}$



$\frac{1}{3} + \frac{1}{4} \rightarrow \frac{(\text{▨} + \text{▨})}{12} = \frac{(4 + 3)}{12} = \frac{7}{12}$

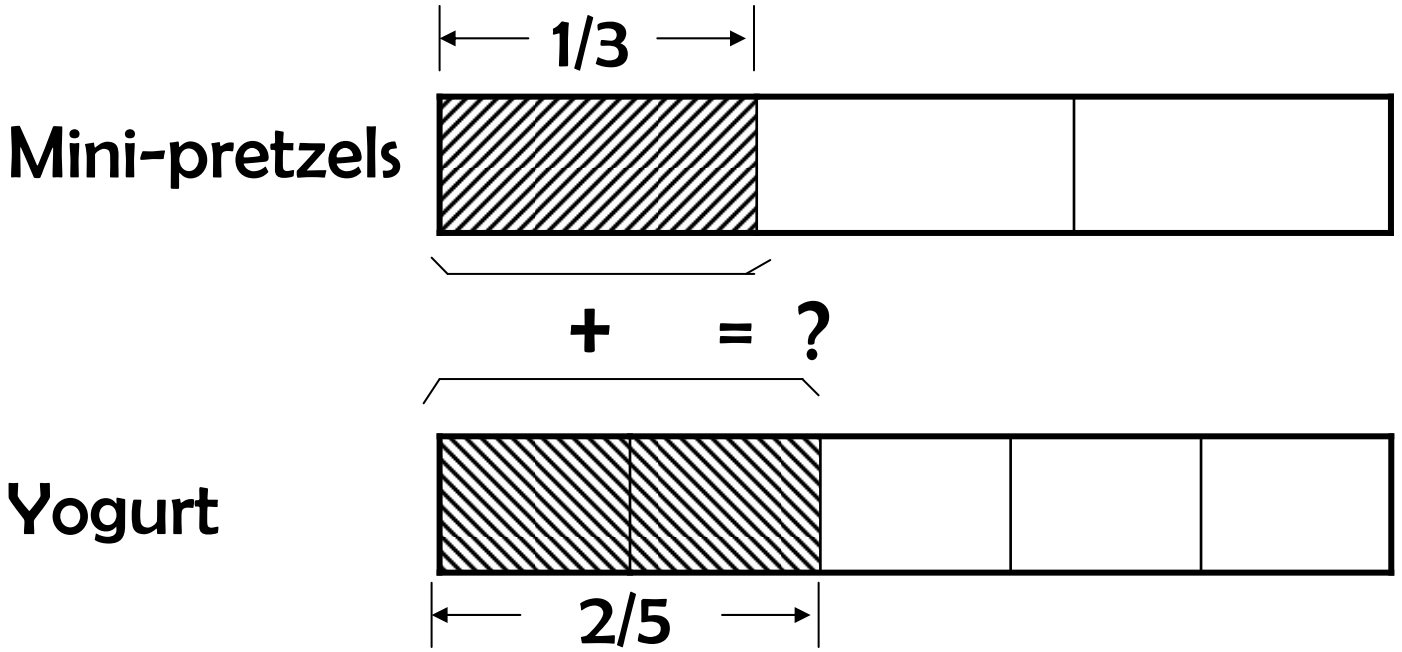
Note: The hatched squares (overlap) are common to each fraction and would therefore be added twice.

Using a day instead of a foot

- $\frac{1}{3}$ of a day is 8 hours and $\frac{1}{4}$ of a day is 6 hours. The sum is 14 hours.
- Because a day contains 24 hours, the answer can be expressed as 14 parts out of 24.
- However, as 14 and 24 are even number, we can simplify to $\frac{7}{12}$.

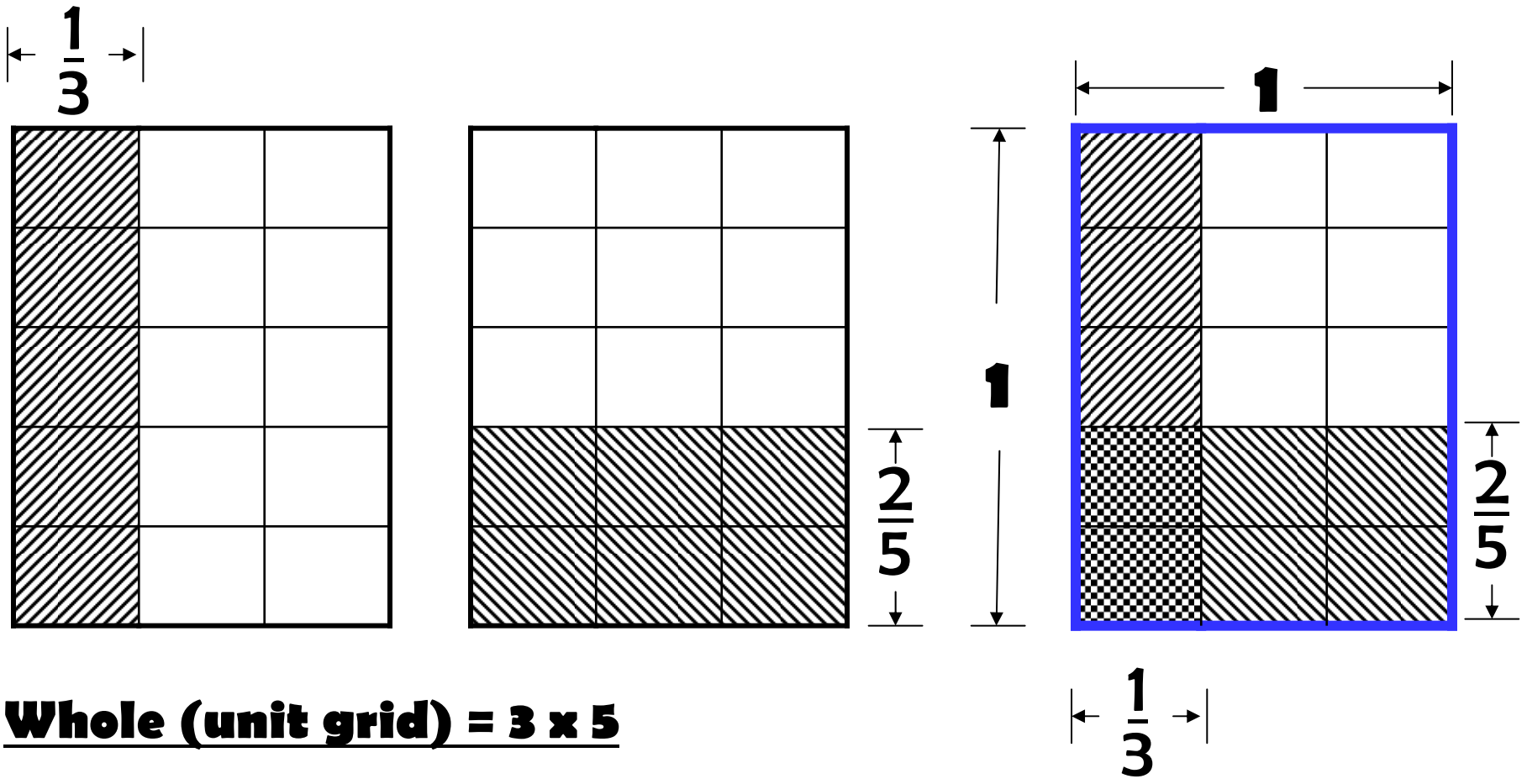
Another Example

Mike likes to make healthy snack. His recipe mixes 1/3 cups of mini-pretzels with 2/5 cups of yogurt-covered cranberries. How much snack does this recipe make?



Addition Fraction with Grid Model

$\frac{1}{3} + \frac{2}{5}$



Whole (unit grid) = 3 x 5

$\frac{1}{3} + \frac{2}{5} \rightarrow \frac{(\text{▨} + \text{▨})}{15} = \frac{(5 + 6 = 11}{15}$ $\frac{11}{15}$

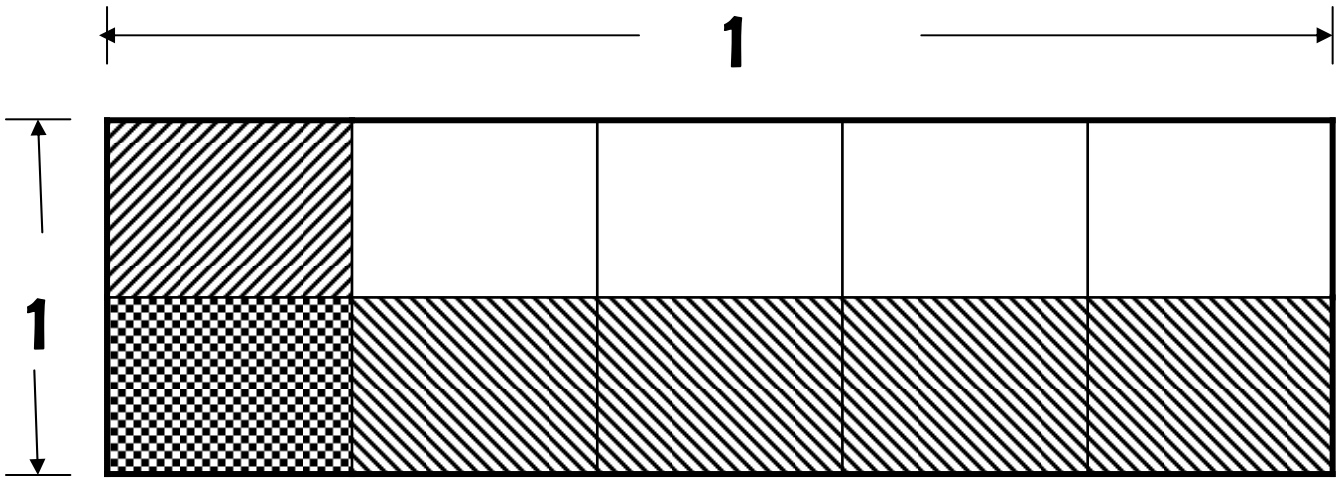
Addition Fraction (2/5 + 1/3)

1. Create a 3 by 5 grid and we define as our *Unit Grid*.
2. Notice that there are 15 small squares in the *Unit Grid*. The denominators show us how to create the *Unit Grid*, while the numerators show us how much of the Unit Grid to shade.
3. One-thirds of the *Unit Grid* (5 squares) have been shaded with lines going from the bottom left to the top right: 
4. Two-fifths of the *Unit Grid* (6 squares) have been shaded with lines going from the top left to the bottom right: 
5. The total number of squares is 5 + 6 = 11, out of a unit number of 15. In other words 10/15, is the answer. Thus: 5/15 + 6/15= 11/15, which is 1/3 + 2/5.

A Different Model for Adding Fractions

Sarah and Cindy won the Math Project. Sarah worked 1/5 hour on their project, and Cindy worked ½ hour of the project. How much time they work together?

Adding Fractions with Grid Model

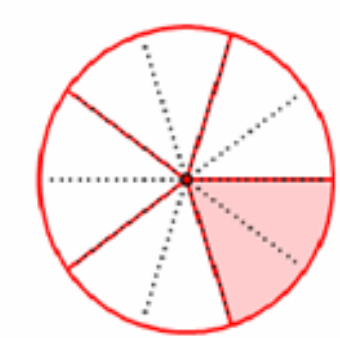
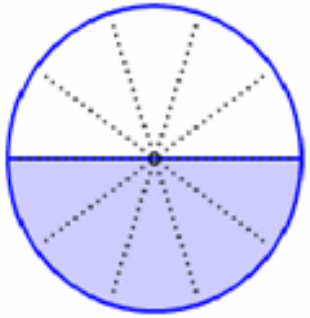
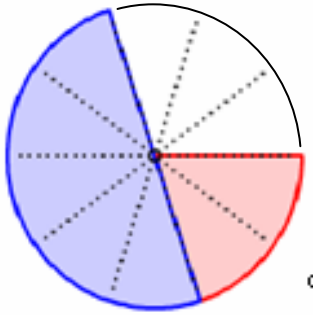


Sarah = $\frac{1}{5}$

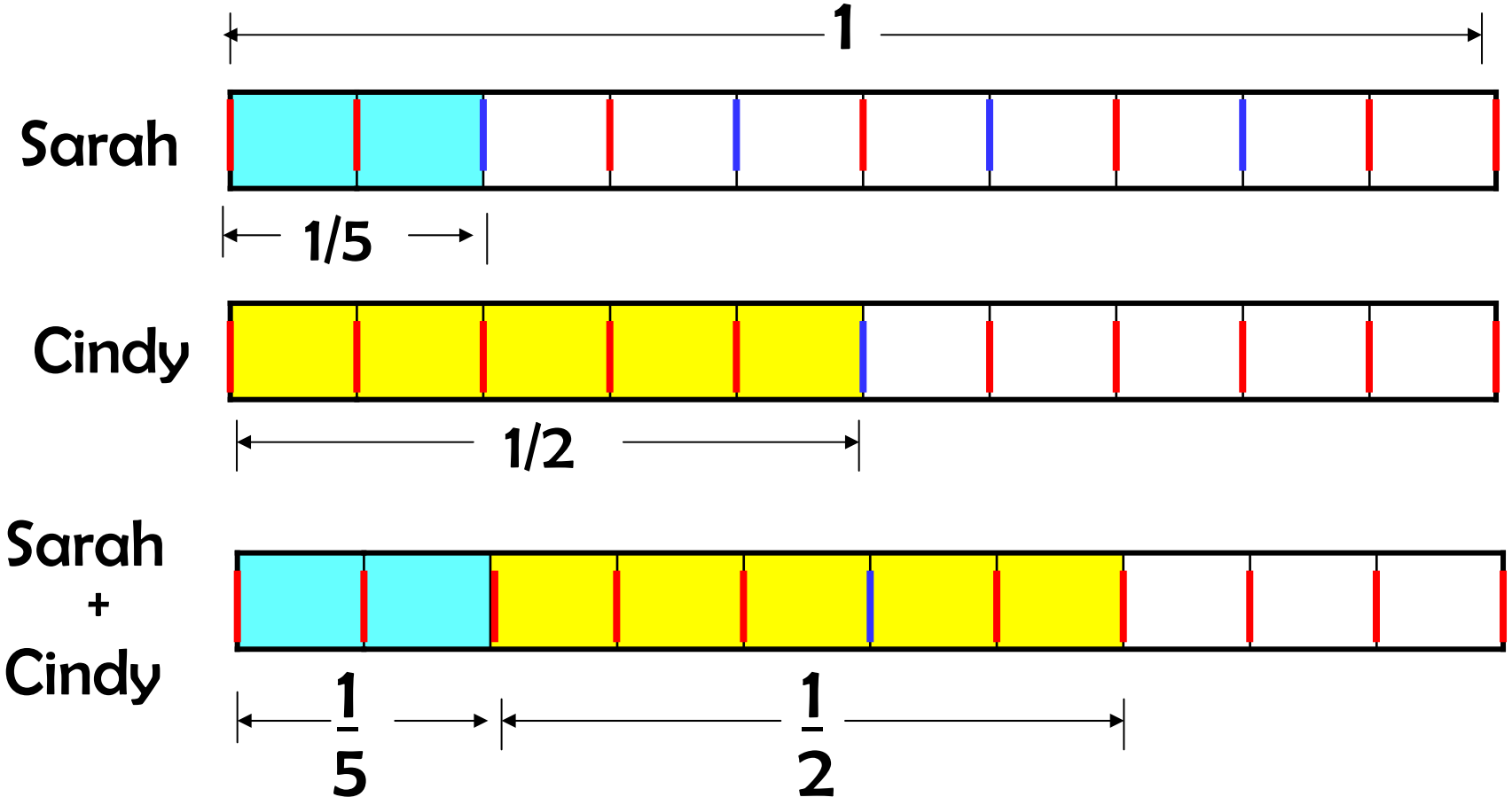
$$\frac{\square + \square}{10} = \frac{2+5}{10} = \frac{7}{10}$$

$$\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$$

Adding Fractions with Circle Model

Sarah		Cindy		Sarah and Cindy
	+		=	
$\frac{1}{5}$	+	$\frac{1}{2}$	=	$\frac{7}{10}$
$\frac{2}{10}$	+	$\frac{5}{10}$	=	$\frac{7}{10}$

Adding Fractions with Bar Diagrams Model



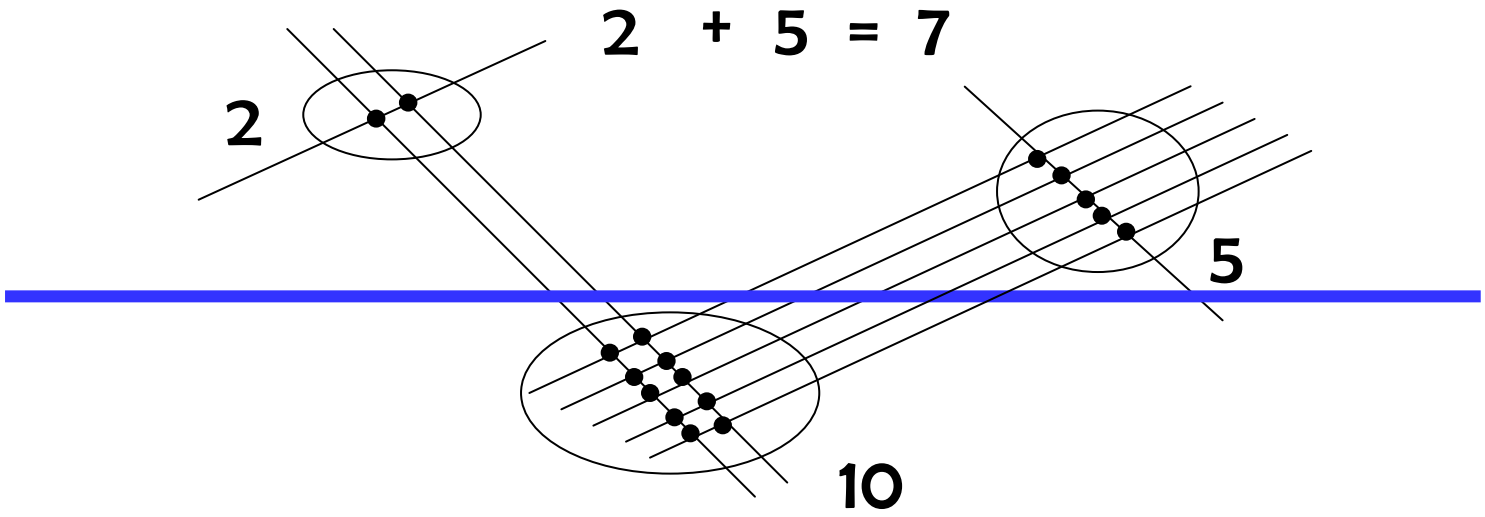
$$\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$$

Adding Fractions with Line Model

$$\frac{a}{b} + \frac{c}{d} = \frac{a \bullet d + b \bullet c}{b \bullet d}$$

$$\frac{1}{5} + \frac{1}{2} = \frac{1 \bullet 2 + 5 \bullet 1}{5 \bullet 2} = \frac{2 + 5}{10}$$

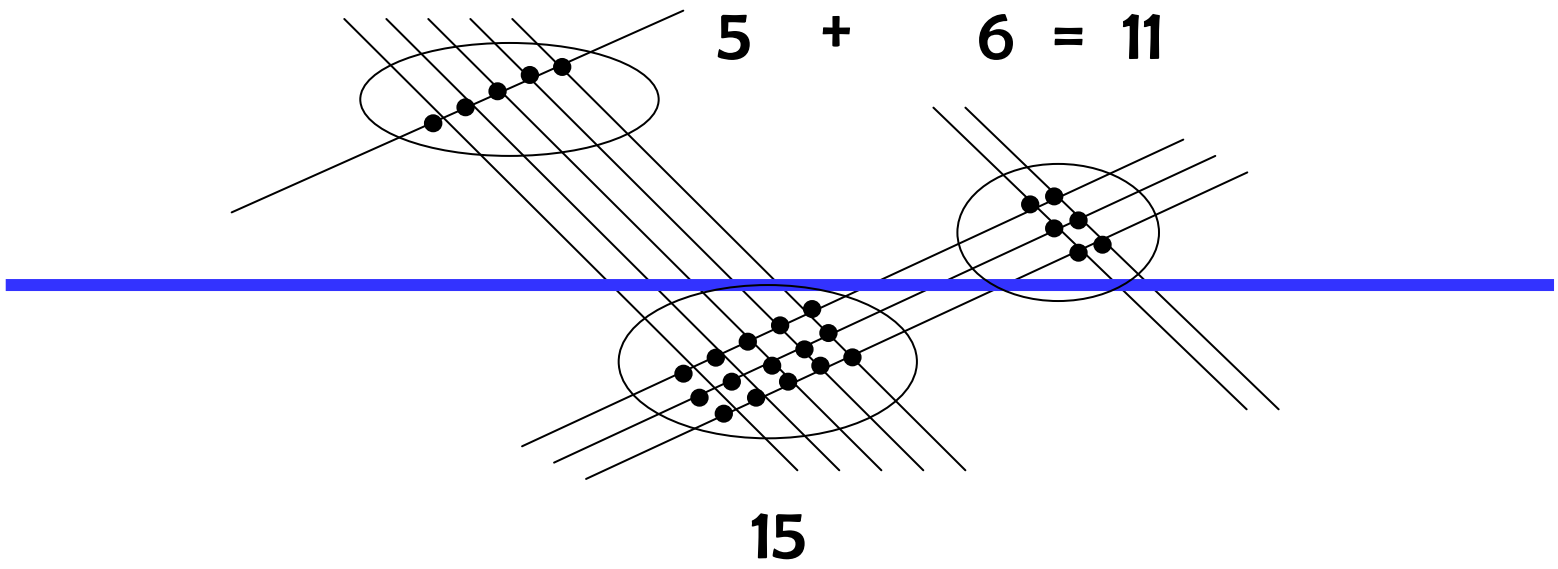
$$\frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$



$$\frac{a}{b} + \frac{c}{d} = \frac{a \bullet d + b \bullet c}{b \bullet d}$$

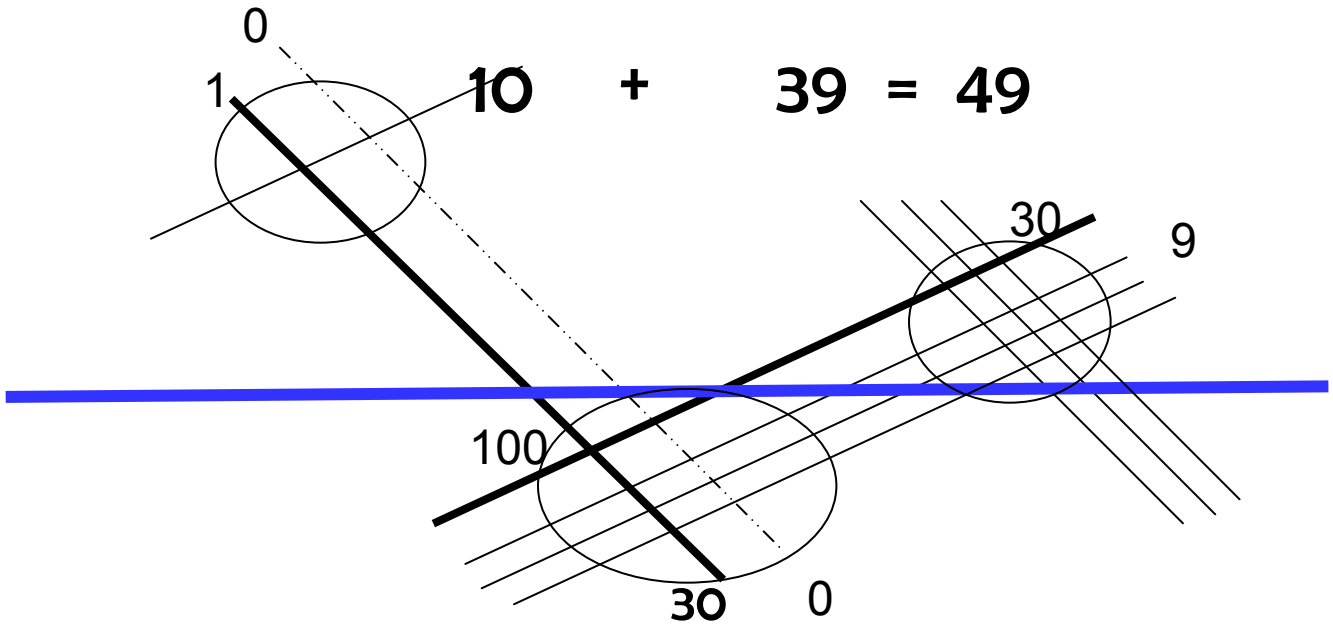
$$\frac{1}{3} + \frac{2}{5} = \frac{1 \bullet 5 + 3 \bullet 2}{3 \bullet 5} = \frac{5 + 6}{15}$$

$$\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$$



Stand for o

$$\frac{1}{13} + \frac{3}{10} = \frac{49}{130}$$



Addition and Subtraction Fraction

If the fractions have different denominators:

1) First, find the least common denominator.
2) Then write equivalent fractions using this denominator.
3) Add or subtract the fractions. Reduce if necessary.

Example:

- 3/4 + 1/6 = ?
- The least common denominator is 12.
- 3/4 + 1/6 = 9/12 + 2/12 = 11/12.

Introduce Subtraction Fraction

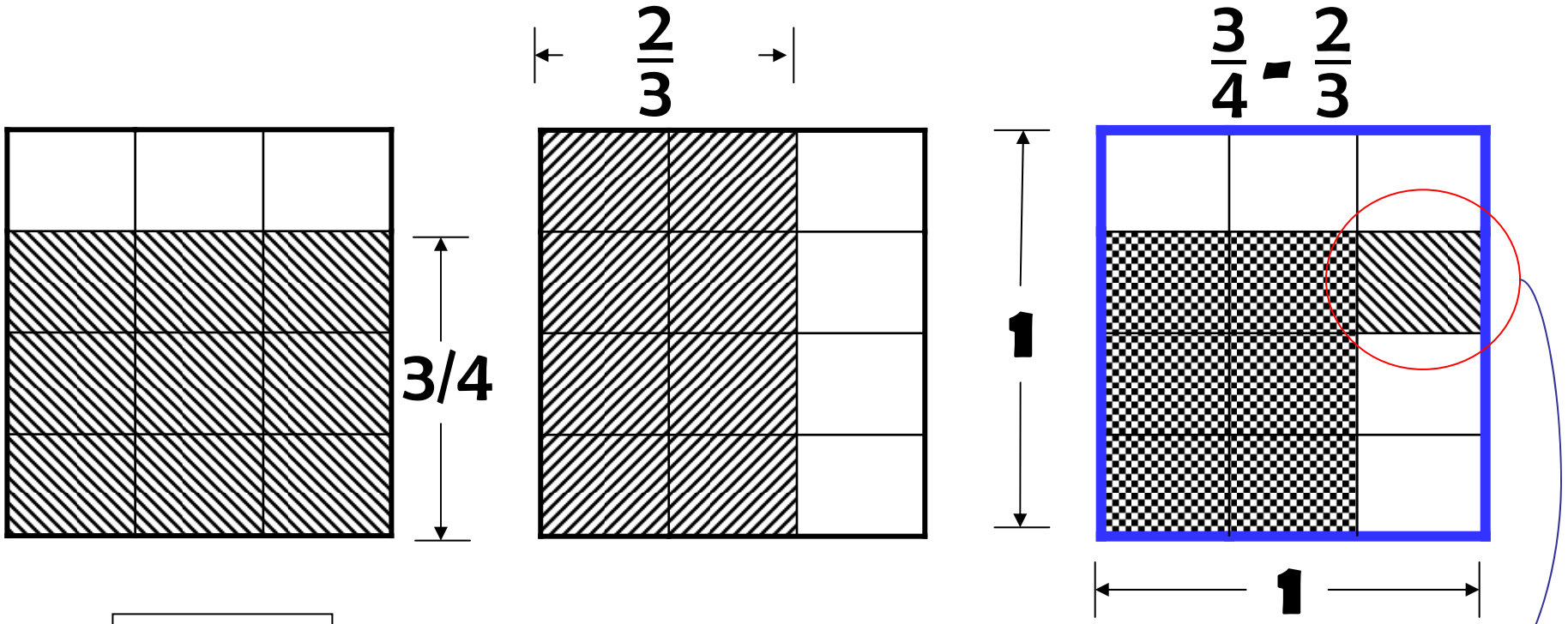
Which is bigger 2/3 or 3/4?

- Relate the fractions to something that breaks easily into thirds or quarters such as a foot, a day, or money.
- Consider a foot. $\frac{2}{3}$ of a foot is 8 inches and $\frac{3}{4}$ of a foot is 9 inches. (one foot = 12 inches)
- So, 9 inches exceeds 8 inches by 1 inch.
- Now, we can subtract the fractions

$$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$$

Subtraction of Fractions with Grid Model

Define a Unit Grid (whole) that is 4 by 3, so there are 12 equal squares. Find how many squares are represented by the two fractions and then subtract. Ignore the hatched squares completely, looking only at the remaining squares.



$$\frac{3}{4} - \frac{2}{3} \rightarrow = \frac{9}{12} - \frac{8}{12} = \frac{(\blacksquare - \blacksquare)}{12} = \frac{(9 - 8)}{12} = \frac{1}{12}$$

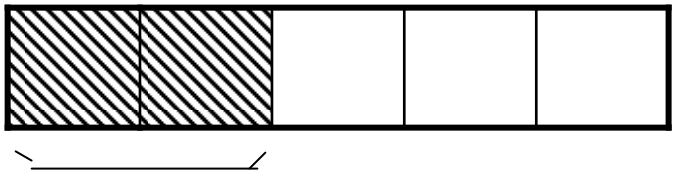
Summary

- We need to make sure that we're teaching fractions **conceptually**, not just algorithmically.
- We need to make sure that we are clearly "defining the whole."
- We can use grid, circle, bar diagrams or line to model addition and subtraction of fractions.

Add Mixed Numbers with Unlike Denominator

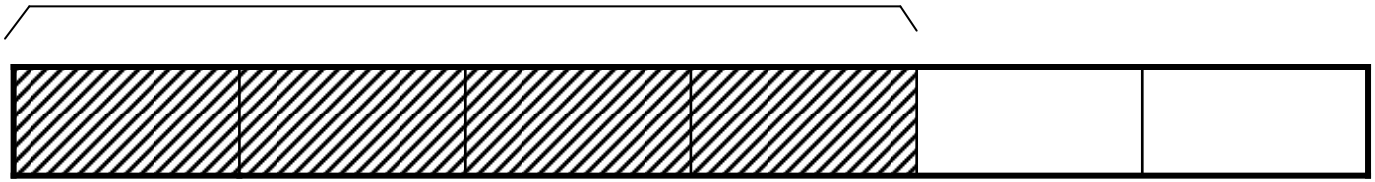
Lily practice for a swim meet by warming up for $\frac{2}{5}$ hours, swimming slow laps for $1\frac{1}{3}$ hours. How long is her practice?

Warming up



\+ = ?

Slow laps

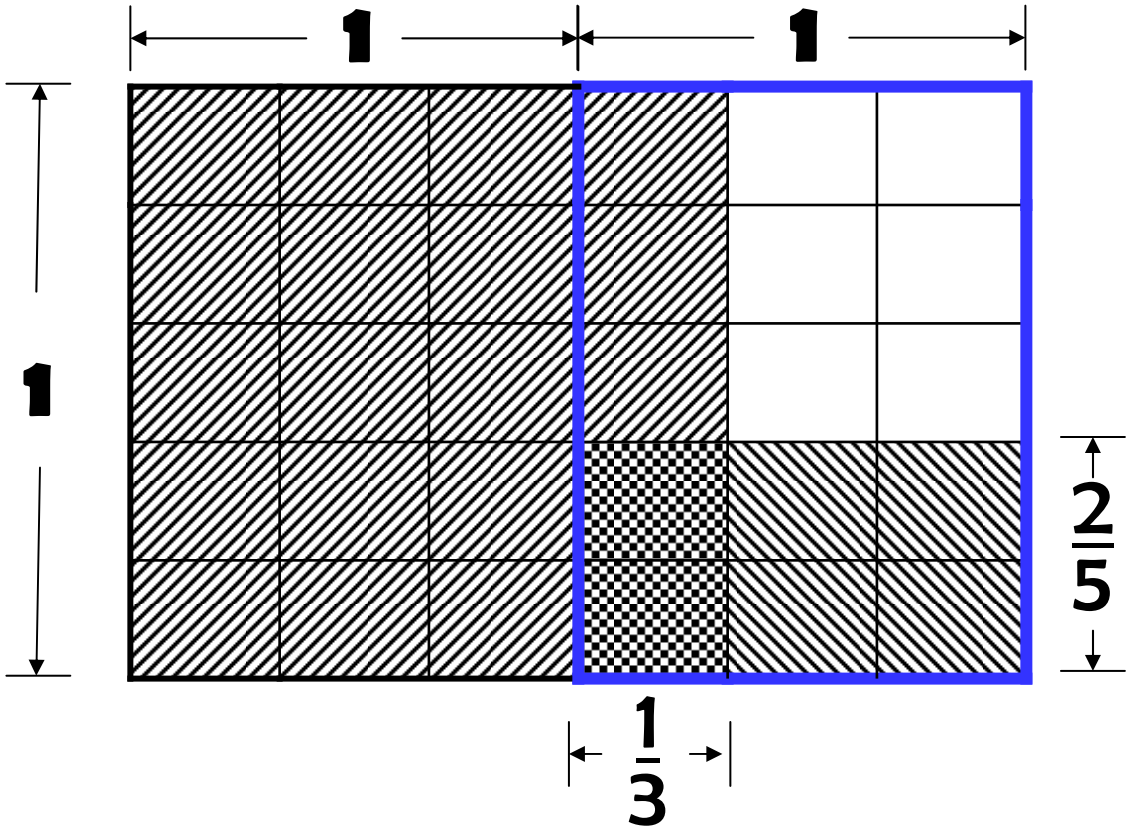


Addition of Mixed Fraction (Improper)

$1\frac{1}{3} + \frac{2}{5}$

$\frac{4}{3} + \frac{2}{5}$

$\frac{20}{15} + \frac{6}{15}$

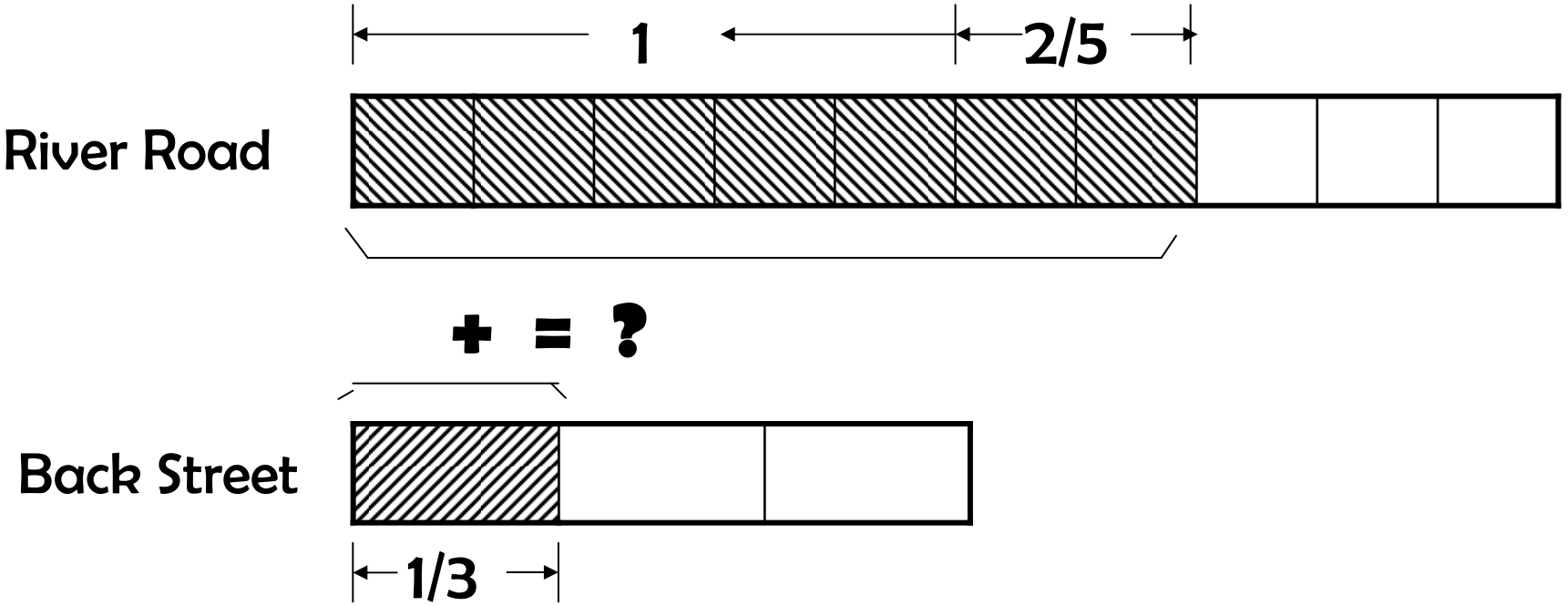


$1\frac{1}{3} + \frac{2}{5}$ → (▨ + ▨) / 15

= {(15+5) + (6)}/15 = 26/15

Add Mixed Numbers with Unlike Denominator

Many runs the same route each day. It's $\frac{1}{3}$ miles on River Road, $1\frac{2}{5}$ miles on Back Street. How far does Many run each day?



Mixed Fraction

$$\frac{1}{3} + 1\frac{2}{5}$$

$$\frac{1}{3} + \frac{7}{5}$$

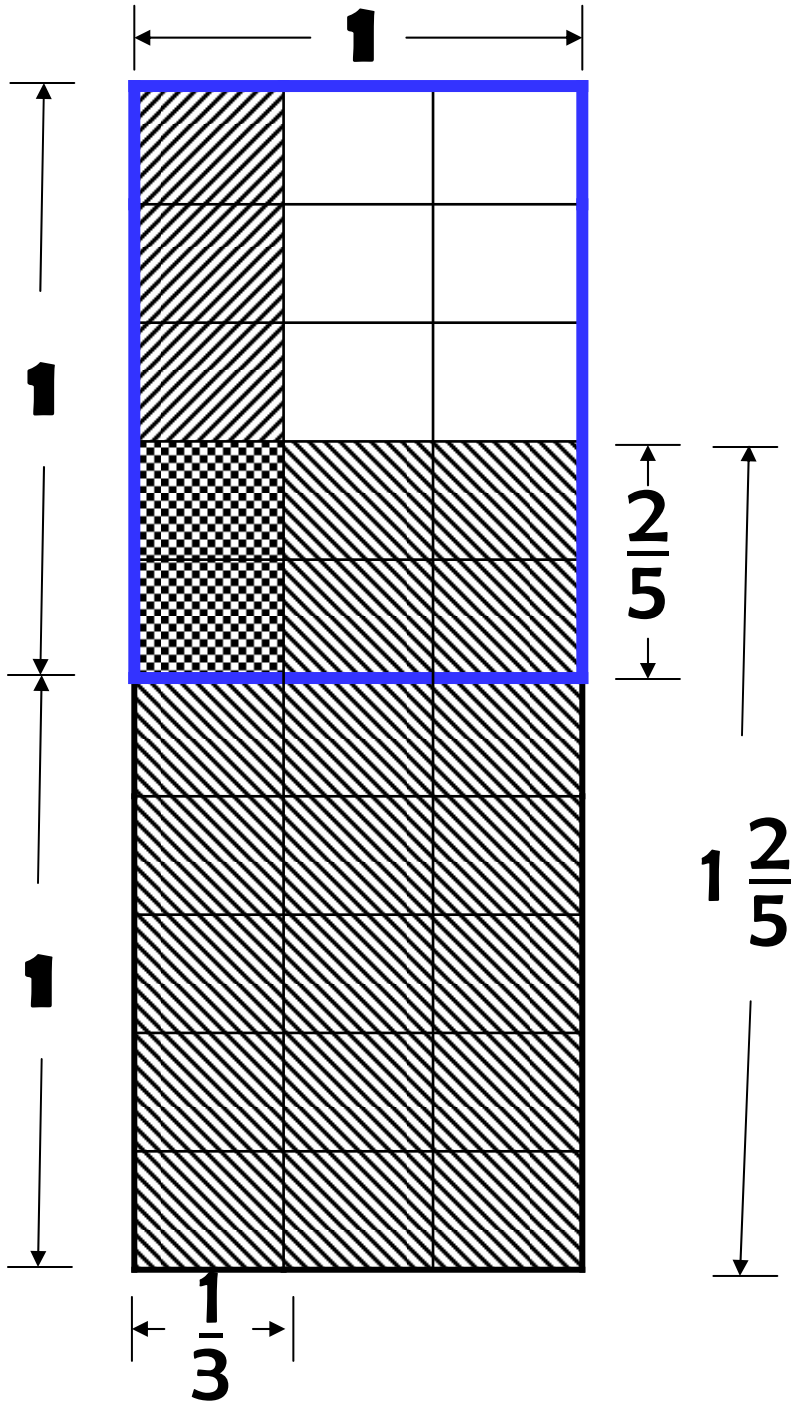
$$\frac{5}{15} + \frac{21}{15}$$

= (▨ + ▨) / 15

= {(5) + (15 +6)} / 15

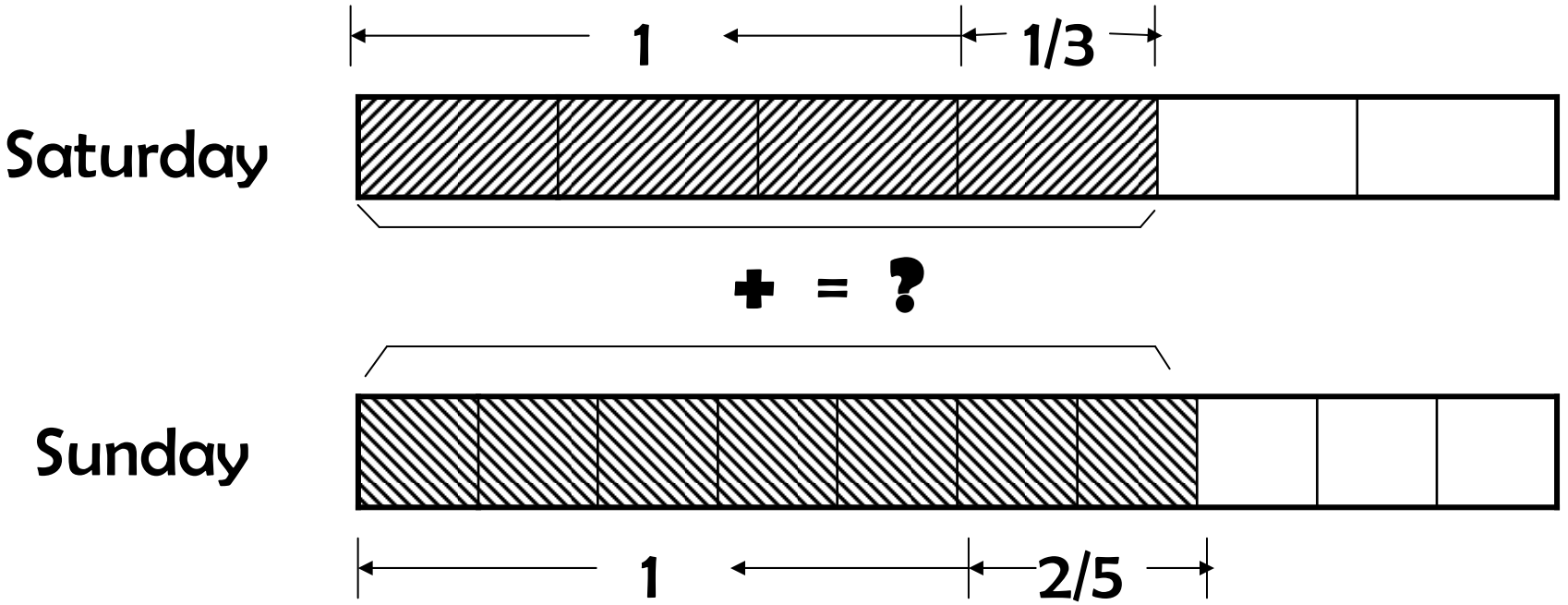
= (5 + 21) / 15

= 26/15



Add Mixed Numbers with Unlike Denominator

Hugo jogged $1\frac{1}{3}$ miles on Saturday and $1\frac{2}{5}$ miles on Sunday. Find the total distance he jogged on those two days



Mixed Fraction

$$1\frac{1}{3} + 1\frac{2}{5}$$

$$\frac{4}{3} + \frac{7}{5}$$

$$\frac{20}{15} + \frac{21}{15}$$

= (▨ + ▨) / 15

= {(15+5) + (15+6)}/15

= (20 + 21)/15

= 41/15